Detailed marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	$\bullet^1$ find mid-point of PQ	$\bullet^1$ $(1,2)$	**3**
	$\bullet^2$ find gradient of median	$\bullet^2$ 2	
	$\bullet^3$ determine equation of median	$\bullet^3$ $y=2x$	

Notes:

1. $\bullet^2$ is only available to candidates who use a midpoint to find a gradient.
2. $\bullet^3$ is only available as a consequence of using the mid-point and the point R, or any other point which lies on the median, eg $(2,4)$.
3. At $\bullet^3$ accept any arrangement of a candidate's equation where constant terms have been simplified.
4. $\bullet^3$ is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A - Perpendicular Bisector of PQ		**Candidate B - Altitude through R**	
$M_{PQ}(1,2)$	$\bullet^1$ ✓	$m_{PQ}=-\frac{2}{3}$	$\bullet^1$ ^
$m_{PQ}=-\frac{2}{3} \Rightarrow m_{\perp}=\frac{3}{2}$	$\bullet^2$ ✗	$m_{\perp}=\frac{3}{2}$	$\bullet^2$ ✗
$2y=3x+1$	$\bullet^3$ $\boxed{\checkmark\ 2}$	$2y=3x+3$	$\bullet^3$ $\boxed{\checkmark\ 2}$
For other perpendicular bisectors award 0/3			

Candidate C - Median through P		**Candidate D - Median through Q**	
$M_{QR}(3\cdot5,3)$	$\bullet^1$ ✗	$M_{PR}(0\cdot5,5)$	$\bullet^1$ ✗
$m_{PM}=-\frac{2}{11}$	$\bullet^2$ $\boxed{\checkmark\ 1}$	$m_{QM}=-\frac{10}{7}$	$\bullet^2$ $\boxed{\checkmark\ 1}$
$11y+2x=40$	$\bullet^3$ $\boxed{\checkmark\ 2}$	$7y+10x=40$	$\bullet^3$ $\boxed{\checkmark\ 2}$

Question	Generic scheme	Illustrative scheme	Max mark
2.	**Method 1**	**Method 1**	3
	$\bullet^1$ equate composite function to x	$\bullet^1$ $g\left(g^{-1}(x)\right)=x$	
	$\bullet^2$ write $g(g^{-1}(x))$ in terms of $g^{-1}(x)$	$\bullet^2$ $\frac{1}{5}g^{-1}(x)-4=x$	
	$\bullet^3$ state inverse function	$\bullet^3$ $g^{-1}(x)=5(x+4)$	
	Method 2	**Method 2**	
	$\bullet^1$ write as $y=\frac{1}{5}x-4$ and start to rearrange	$\bullet^1$ $y+4=\frac{1}{5}x$	
	$\bullet^2$ express x in terms of y	$\bullet^2$ eg $x=5(y+4)$ or $x=\frac{(y+4)}{\frac{1}{5}}$	
	$\bullet^3$ state inverse function	$\bullet^3$ $g^{-1}(x)=5(x+4)$	
	Method 3	**Method 3**	
	$\bullet^1$ interchange variables	$\bullet^1$ $x=\frac{1}{5}y-4$	
	$\bullet^2$ express y in terms of x	$\bullet^2$ eg $y=5(x+4)$ or $y=\frac{(x+4)}{\frac{1}{5}}$	
	$\bullet^3$ state inverse function	$\bullet^3$ $g^{-1}(x)=5(x+4)$	

Notes:

1. $y=5(x+4)$ does not gain $\bullet^3$.
2. At $\bullet^3$ stage, accept g^{-1} written in terms of any dummy variable eg $g^{-1}(y)=5(y+4)$.
3. $g^{-1}(x)=5(x+4)$ with no working gains 3/3.

Commonly Observed Responses:

Candidate A

$x \rightarrow \frac{1}{5}x \rightarrow \frac{1}{5}x-4=g(x)$

$\div 5 \rightarrow -4$

$\therefore\ +4 \rightarrow \times 5$ — $\bullet^1$ ✓ awarded for knowing to perform inverse operations in reverse order

$5(x+4)$ — $\bullet^2$ ✓

$g^{-1}(x)=5(x+4)$ — $\bullet^3$ ✓

Candidate B - BEWARE		Candidate C	
$g'(x)=\ldots$	$\bullet^3$ ✗	$g^{-1}(x)=5x+4$	**with no working Award 0/3**

Question	Generic scheme	Illustrative scheme	Max mark
3.	$\bullet^1$ start to differentiate $\bullet^2$ complete differentiation $\bullet^3$ evaluate derivative	$\bullet^1$ $-3\sin 2x\ldots$ stated or implied by $\bullet^2$ $\bullet^2$ $\ldots\times 2$ $\bullet^3$ $-3\sqrt{3}$	3

Notes:

1. Ignore the appearance of $+c$ at any stage.
2. $\bullet^3$ is available for evaluating an attempt at finding the derivative at $\frac{\pi}{6}$.
3. For $h'\left(\frac{\pi}{6}\right)=3\cos\left(2\times\frac{\pi}{6}\right)=\frac{3}{2}$ award 0/3.

Commonly Observed Responses:

Candidate A		Candidate B		Candidate C	
$-3\sin 2x\ldots$	$\bullet^1$ ✓	$3\sin 2x\ldots$	$\bullet^1$ ✗	$3\sin 2x\ldots$	$\bullet^1$ ✗
$\ldots\times\frac{1}{2}$	$\bullet^2$ ✗	$\ldots\times 2$	$\bullet^2$ ✓	$\ldots\times\frac{1}{2}$	$\bullet^2$ ✗
$-\frac{3\sqrt{3}}{4}$	$\bullet^3$ $\boxed{✓ 1}$	$3\sqrt{3}$	$\bullet^3$ $\boxed{✓ 1}$	$\frac{3\sqrt{3}}{4}$	$\bullet^3$ $\boxed{✓ 1}$
Candidate D		**Candidate E**		**Candidate F**	
$\pm 6\cos 2x$	$\bullet^1$ ✗	$\pm 3\cos 2x\ldots$	$\bullet^1$ ✗	$6\sin 2x$	$\bullet^1$ ✗
	$\bullet^2$ ✗	$\ldots\times 2$	$\bullet^2$ $\boxed{✓ 1}$		$\bullet^2$ ✓
± 3	$\bullet^3$ $\boxed{✓ 1}$	± 3	$\bullet^3$ $\boxed{✓ 1}$	$3\sqrt{3}$	$\bullet^3$ $\boxed{✓ 1}$

Question	Generic scheme	Illustrative scheme	Max mark
4.	$\bullet^1$ state centre of circle $\bullet^2$ find gradient of radius $\bullet^3$ state gradient of tangent $\bullet^4$ state equation of tangent	$\bullet^1$ $(6,3)$ $\bullet^2$ -4 $\bullet^3$ $\frac{1}{4}$ $\bullet^4$ $y=\frac{1}{4}x-7$	**4**
Notes:			
1. Accept $-\frac{8}{2}$ for $\bullet^2$. 2. The perpendicular gradient must be simplified at the $\bullet^3$ or $\bullet^4$ stage for $\bullet^3$ to be available. 3. $\bullet^4$ is only available as a consequence of trying to find and use a perpendicular gradient. 4. At $\bullet^4$ accept $y-\frac{1}{4}x+7=0$, $4y=x-28$, $x-4y-28=0$ or any other rearrangement of the equation where the constant terms have been simplified.			
Commonly Observed Responses:			

Question	Generic scheme	Illustrative scheme	Max mark
5. (a)	•[1] state ratio explicitly	•[1] 4 : 1	**1**
Notes:			
1. The only acceptable variations for •[1] must be related explicitly to AB and BC. For $\frac{BC}{AB}=\frac{1}{4}$, $\frac{AB}{BC}=\frac{4}{1}$ or $BC:AB=1:4$ award 1/1. 2. For $BC=\frac{1}{4}AB$ award 0/1.			
Commonly Observed Responses:			
(b)	•[2] state value of t	•[2] 8	**1**
Notes:			
3. The answer to part (b) **must** be consistent with a ratio stated in part (a) unless a valid strategy which does not require the use of their ratio from part (a) is used.			
Commonly Observed Responses:			

Candidate A		**Candidate B**	
$1:4$	•[1] ✗	$1:4$	•[1] ✗
$t=8$	•[2] ✗	$t=5$	•[2] $\boxed{✓ 1}$

Question	Generic scheme	Illustrative scheme	Max mark
6.	$\bullet^1$ apply $m\log_5 x = \log_5 x^m$ $\bullet^2$ apply $\log_5 x - \log_5 y = \log_5 \frac{x}{y}$ $\bullet^3$ evaluate log	$\bullet^1$ $\log_5 8^{\frac{1}{3}}$ $\bullet^2$ $\log_5 \left(\frac{250}{8^{\frac{1}{3}}} \right)$ $\bullet^3$ 3	3

Notes:

1. Each line of working must be equivalent to the line above within a valid strategy, however see Candidate B.
2. Do not penalise the omission of the base of the logarithm at $\bullet^1$ or $\bullet^2$.
3. For '3' with no working award 0/3.

Commonly Observed Responses:

Candidate A		Candidate B	
$\log_5 250 - \log_5 \frac{8}{3}$	$\bullet^1$ ✗	$\frac{1}{3}\log_5 (250 \div 8)$	
$\log_5 \frac{250}{\frac{8}{3}}$	$\bullet^2$ $\boxed{✓ 1}$	$\frac{1}{3}\log_5 \frac{125}{4}$	
$\log_5 \frac{375}{4}$	$\bullet^3$ $\boxed{✓ 2}$	$\log_5 \left(\frac{125}{4}\right)^{\frac{1}{3}}$	Award 1/3 $\boxed{✓ 1}$ ✗ ^
			$\bullet^1$ is awarded for the final two lines of working

Question		Generic scheme	Illustrative scheme	Max mark
7.	(a)	•[1] state coordinates of P	•[1] $(0,5)$	**1**

Notes:

1. Accept '$x=0,\ y=5$'.
2. '$y=5$' alone or '5' does not gain •[1].

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	•[2] differentiate •[3] calculate gradient •[4] state equation of tangent	•[2] $3x^2-6x+2$ •[3] 2 •[4] $y=2x+5$	**3**

Notes:

3. At •[4] accept $y-2x=5$, $2x-y+5=0$, $y-5=2x$ or any other rearrangement of the equation where the constant terms have been simplified.
4. •[4] is only available if an attempt has been made to find the gradient from differentiation.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
7.	(c)	•5 set $y_{\text{line}} = y_{\text{curve}}$ and arrange in standard form	•5 $x^3 - 3x^2 = 0$	4
		•6 factorise	•6 $x^2(x-3)$	
		•7 state x-coordinate of Q	•7 3	
		•8 calculate y-coordinate of Q	•8 11	

Notes:

5. •5 is only available if '$=0$' appears at either •5 or •6 stage.
6. •7 and •8 are only available as a consequence of solving a cubic equation and a linear equation simultaneously.
7. For an answer of $(3,11)$ with no working award 0/4.
8. For an answer of $(3,11)$ verified in both equations award 3/4.
9. For an answer of $(3,11)$ verified in both equations along with a statement such as 'same point on both line and curve so Q is $(3,11)$' award 4/4.
10. For candidates who work with a derivative, no further marks are available.
11. $x=3$ must be supported by valid working for •7 and •8 to be awarded.

Commonly Observed Responses:

Candidate A

$x^3 - 3x^2 = 0$	•5 ✓
$x - 3 = 0$	•6 ✓
$x = 3$	•7 ✓
$y = 11$	•8 ✓

Dividing by x^2 is valid since $x \neq 0$ at •6

<table>
<tr><th>Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>8.</td><td>•1 determine the gradient of the line

•2 determine the angle</td><td>•1 $m=\sqrt{3}$ or $\tan\theta=\sqrt{3}$

•2 $60°$ or $\frac{\pi}{3}$</td><td>2</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">1. Do not penalise the omission of units at •2.
2. For $60°$ or $\frac{\pi}{3}$ without working award 2/2.</td></tr>
<tr><td colspan="4">Commonly Observed Responses:</td></tr>
<tr><td colspan="2">Candidate A
$y=\sqrt{3}x+5$ Ignore incorrect processing of the constant term
$m=\sqrt{3}$ •1 ✓
$60°$ •2 ✓</td><td colspan="2">Candidate B
$m=\sqrt{3}$ •1 ✓
$\theta=\tan\sqrt{3}$ •2 ✗
$\theta=60°$
Stating $\tan$ rather than $\tan^{-1}$
See general marking principle (g)</td></tr>
</table>

Question		Generic scheme	Illustrative scheme	Max mark
9.	(a)	$\bullet^1$ identify pathway	$\bullet^1$ $-\mathbf{t}+\mathbf{u}$	**1**

Notes:

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	$\bullet^2$ state an appropriate pathway $\bullet^3$ express pathway in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$	$\bullet^2$ eg $\frac{1}{2}\overrightarrow{BC}+\overrightarrow{CA}+\overrightarrow{AD}$ stated or implied by $\bullet^3$ $\bullet^3$ $-\frac{1}{2}\mathbf{t}-\frac{1}{2}\mathbf{u}+\mathbf{v}$	**2**

Notes:

1. There is no need to simplify the expression at $\bullet^3$. Eg $\frac{1}{2}(-\mathbf{t}+\mathbf{u})-\mathbf{u}+\mathbf{v}$.
2. $\bullet^3$ is only available for using a valid pathway.
3. The expression at $\bullet^3$ must be consistent with the candidate's expression at $\bullet^1$.
4. If the pathway in $\bullet^1$ is given in terms of a single vector $\mathbf{t}$, $\mathbf{u}$ or $\mathbf{v}$, then $\bullet^3$ is not available.

Commonly Observed Responses:

Candidate A $\overrightarrow{MD}=-\frac{1}{2}\mathbf{t}+\mathbf{v}-\mathbf{u}$ $\quad\bullet^2$ ^ $\bullet^3$ ✗	

Question	Generic scheme	Illustrative scheme	Max mark
10.	•¹ know to and integrate one term •² complete integration •³ substitute for x and y •⁴ state equation	•¹ eg $2x^3\ldots$ •² eg $\ldots -\frac{3}{2}x^2+4x+c$ •³ $14=2(2)^3-\frac{3}{2}(2)^2+4(2)+c$ •⁴ $y=2x^3-\frac{3}{2}x^2+4x-4$ stated explicitly	4

Notes:

1. For candidates who make no attempt to integrate to find y in terms of x award 0/4.
2. For candidates who omit $+c$, only •¹ is available.
3. Candidates must attempt to integrate both terms containing x for •³ and •⁴ to be available. See Candidate B.
4. For candidates who differentiate any term, •² •³ and •⁴ are not available.
5. •⁴ is not available for '$f(x)=\ldots$'.
6. Candidates must simplify coefficients in **<u>their</u>** final line of working for the last mark available in that line of working to be awarded.

Commonly Observed Responses:

Candidate A		Candidate B - partial integration	
$y=2x^3-\frac{3}{2}x^2+4x+c$	•¹ ✓ •² ✓	$y=2x^3-\frac{3}{2}x^2+4+c$	•¹ ✓ •² ✗
$y=2(2)^3-\frac{3}{2}(2)^2+4(2)+c$		$14=2(2)^3-\frac{3}{2}(2)^2+4+c$	•³ $\boxed{✓ 1}$
$c=-4$	•³ ✓ substitution for y implied by $c=-4$ •⁴ ^	$c=0$	
		$y=2x^3-\frac{3}{2}x^2+4$	•⁴ $\boxed{✓ 1}$

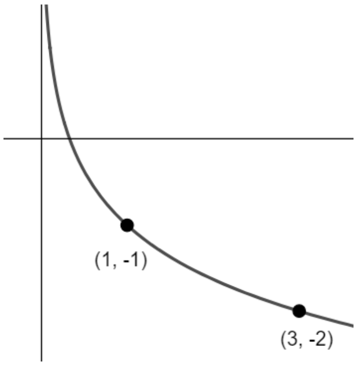
Question	Generic scheme	Illustrative scheme	Max mark
11. (a)	•¹ curve reflected in x-axis and translated 1 unit vertically •² accurate sketch	•¹ a generally decreasing curve above the x-axis for $1 < x < 3$ •² asymptote at $x = 0$ **and** passing through $(3,0)$ **and** continuing to decrease for $x \geq 3$	2

Notes:

1. For any attempt which involves a horizontal translation or reflection in the y-axis award 0/2.
2. For a single transformation award 0/2.
3. For any attempt involving a reflection in the line $y = x$ award 0/2

Commonly Observed Responses:

Candidate A



Award 1/2

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•³ set ‘ $y = y$ ’ •⁴ start to solve •⁵ state x coordinate	•³ $\log_3 x = 1 - \log_3 x$ •⁴ $\log_3 x = \frac{1}{2}$ or $\log_3 x^2 = 1$ •⁵ $\sqrt{3}$ or $3^{\frac{1}{2}}$	3

Notes:

4. •³ may be implied by $\log_3 x = \frac{1}{2}$ from symmetry of the curves.
5. Do not penalise the omission of the base of the logarithm at •³ or •⁴.
6. For a solution which equates a to $\log_3 a$, the final mark is not available.
7. If a candidate considers and then does not discard $-\sqrt{3}$ in their final answer, •⁵ is not available.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
12.	(a)	$\bullet^1$ find components	$\bullet^1$ $\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	**1**

Notes:

1. Accept $6\mathbf{i}-3\mathbf{j}+(4+p)\mathbf{k}$ for $\bullet^1$.
2. Do not accept $\begin{pmatrix} 6\mathrm{i} \\ -3\mathbf{j} \\ (4+p)\mathbf{k} \end{pmatrix}$ or $6\mathbf{i}-3\mathbf{j}+4\mathbf{k}+p\mathbf{k}$ for $\bullet^1$. However $\bullet^2$, $\bullet^3$ and $\bullet^4$ are still available.

Commonly Observed Responses:

Question		Generic scheme	Illustrative scheme	Max mark
	(b)	$\bullet^2$ find an expression for magnitude	$\bullet^2$ $\sqrt{6^2+(-3)^2+(4+p)^2}$	**3**
		$\bullet^3$ start to solve	$\bullet^3$ $45+(4+p)^2=49 \Rightarrow (4+p)^2=4$ or $p^2+8p+12=0$	
		$\bullet^4$ find values of p	$\bullet^4$ $p=-2,\ p=-6$	

Notes:

3. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. Eg $\sqrt{6^2+-3^2+(4+p)^2}$ or $\sqrt{6^2-3^2+(4+p)^2}$ leading to $\sqrt{45+(4+p)^2}$, $\bullet^2$ is awarded.
4. $\bullet^4$ is only available for two distinct values of p.

Commonly Observed Responses:

Candidate A		**Candidate B**	
$\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	$\bullet^1$ ✓	$\begin{pmatrix} 6 \\ -3 \\ 4+p \end{pmatrix}$	$\bullet^1$ ✓
$\sqrt{6^2-3^2+(4+p)^2}$	$\bullet^2$ ✗	$\sqrt{6^2+(-3)^2+p^2}$	$\bullet^2$ ✗
$27+(4+p)^2=49$		$45+p^2=49$	$\bullet^3$ $\boxed{✓\ 2}$
$(4+p)^2=22$	$\bullet^3$ $\boxed{✓\ 1}$	$p=\pm 2$	$\bullet^4$ $\boxed{✓\ 1}$
$p=-4\pm\sqrt{22}$	$\bullet^4$ $\boxed{✓\ 1}$		

Question	Generic scheme	Illustrative scheme	Max mark
13. (a) (i)	$\bullet^1$ find the value of $\cos x$ $\bullet^2$ substitute into the formula for $\sin 2x$ $\bullet^3$ simplify	$\bullet^1$ $\frac{\sqrt{7}}{\sqrt{11}}$ stated or implied by $\bullet^2$ $\bullet^2$ $2\times\frac{2}{\sqrt{11}}\times\frac{\sqrt{7}}{\sqrt{11}}$ $\bullet^3$ $\frac{4\sqrt{7}}{11}$	3
(ii)	$\bullet^4$ evaluate $\cos 2x$	$\bullet^4$ $\frac{3}{11}$	1

Notes:

1. Where a candidate substitutes an incorrect value for $\cos x$ at $\bullet^2$, $\bullet^2$ may be awarded if the candidate has previously stated this incorrect value or it can be implied by a diagram.
2. $\bullet^3$ is only available as a consequence of substituting into a valid formula at $\bullet^2$.
3. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b)	$\bullet^5$ expand using the addition formula $\bullet^6$ substitute in values $\bullet^7$ simplify	$\bullet^5$ $\sin 2x\cos x+\cos 2x\sin x$ stated or implied by $\bullet^6$ $\bullet^6$ $\frac{4\sqrt{7}}{11}\times\frac{\sqrt{7}}{\sqrt{11}}+\frac{3}{11}\times\frac{2}{\sqrt{11}}$ $\bullet^7$ $\frac{34}{11\sqrt{11}}$	3

Notes:

4. For any attempt to use $\sin(2x+x)=\sin 2x+\sin x$, $\bullet^5$ $\bullet^6$ and $\bullet^7$ are not available

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
14.	$\bullet^1$ write in integrable form $\bullet^2$ start to integrate $\bullet^3$ complete integration $\bullet^4$ process limits $\bullet^5$ evaluate integral	$\bullet^1$ $(2x+9)^{-\frac{2}{3}}$ $\bullet^2$ $\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\ldots$ $\bullet^3$ $\ldots\times\dfrac{1}{2}$ $\bullet^4$ $\dfrac{3}{2}(2(9)+9)^{\frac{1}{3}}-\dfrac{3}{2}(2(-4)+9)^{\frac{1}{3}}$ $\bullet^5$ 3	5

Notes:

1. For candidates who differentiate throughout, only $\bullet^1$ is available.
2. For candidates who 'integrate the denominator' without attempting to write in integrable form award 0/5.
3. $\bullet^2$ may be awarded for the appearance of $\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}$ in the line of working where the candidate first attempts to integrate. See Candidate F.
4. If candidates start to integrate individual terms within the bracket or attempt to expand a bracket or use another invalid approach no further marks are available.
5. For $\bullet^2$ to be awarded the integrand must contain a non-integer power.
6. Do not penalise the inclusion of '$+c$'.
7. $\bullet^4$ and $\bullet^5$ are not available to candidates who substitute into the original function.
8. The integral obtained must contain a non-integer power for $\bullet^5$ to be available.
9. $\bullet^5$ is only available to candidates who deal with the coefficient of x at the $\bullet^3$ stage. See Candidate A.

Commonly Observed Responses:

Candidate A		Candidate B	
$(2x+9)^{-\frac{2}{3}}$	$\bullet^1$ ✓	$(2x+9)^{\frac{2}{3}}$	$\bullet^1$ ✗
$\dfrac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}$	$\bullet^2$ ✓ $\bullet^3$ ^	$\dfrac{(2x+9)^{\frac{5}{3}}}{\frac{5}{3}}\times\dfrac{1}{2}$	$\bullet^2$ $\boxed{✓ 1}$ $\bullet^3$ ✓
$3(2(9)+9)^{\frac{1}{3}}-3(2(-4)+9)^{\frac{1}{3}}$	$\bullet^4$ $\boxed{✓ 1}$	$\dfrac{3}{10}(2(9)+9)^{\frac{5}{3}}-\dfrac{3}{10}(2(-4)+9)^{\frac{5}{3}}$	$\bullet^4$ $\boxed{✓ 1}$
6	$\bullet^5$ $\boxed{✓ 2}$ see note 9	$\dfrac{363}{5}$	$\bullet^5$ $\boxed{✓ 1}$

Commonly Observed Responses:	
Candidate C $(2x+9)^{-\frac{2}{3}}$ •1 ✓ $-\frac{5}{3}(2x+9)^{-\frac{5}{3}}\times\frac{1}{2}$ •2 ✗ •3 ✓ $-\frac{5}{6}(2(9)+9)^{-\frac{5}{3}}-\left(-\frac{5}{6}(2(-4)+9)^{-\frac{5}{3}}\right)$ •4 $\boxed{✓ 1}$ $\frac{605}{729}$ •5 $\boxed{✓ 1}$	**Candidate D** $(2x+9)^{-\frac{2}{3}}$ •1 ✓ $\frac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\times 2$ •2 ✓ •3 ✗ $6(2(9)+9)^{\frac{1}{3}}-6(2(-4)+9)^{\frac{1}{3}}$ •4 $\boxed{✓ 1}$ 12 •5 $\boxed{✓ 1}$
Candidate E $(2x+9)^{-\frac{3}{2}}$ •1 ✗ $\frac{(2x+9)^{-\frac{1}{2}}}{-\frac{1}{2}}\times\frac{1}{2}$ •2 $\boxed{✓ 1}$ •3 ✓ $\left(-(2(9)+9)^{-\frac{1}{2}}\right)-\left(-(2(-4)+9)^{-\frac{1}{2}}\right)$ •4 $\boxed{✓ 1}$ $-\frac{1}{\sqrt{27}}+1$ •5 $\boxed{✓ 1}$	**Candidate F** $1\times(2x+9)^{-\frac{2}{3}}$ •1 ✓ $x\frac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3}}\times\frac{1}{2}$ •2 ✓ •3 •4 and •5 are not available

Question	Generic scheme	Illustrative scheme	Max mark
15.	•1 root at $x = -4$ identifiable from graph •2 stationary point touching x-axis when $x = 2$ identifiable from graph •3 stationary point when $x = -2$ identifiable from graph •4 identify orientation of the cubic curve and $f'(0) > 0$ identifiable from graph	•1 •2 •3 •4 	4

Notes:

1. For a diagram which does not show a cubic curve award 0/4.
2. For candidates who identify the roots of the cubic at '$x = -4, -2$ and 2' or at '$x = -2, 2$ and 4' •4 is unavailable.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

National
Qualifications
2018

2018 Mathematics

Higher - Paper 2

Finalised Marking Instructions

Detailed marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	•¹ state an integral to represent the shaded area	•¹ $\int_{-1}^{3}\left(3+2x-x^2\right)dx$	4
	•² integrate	•² $3x+\frac{2x^2}{2}-\frac{x^3}{3}$	
	•³ substitute limits	•³ $\left(3\times3+\frac{2\times3^2}{2}-\frac{3^3}{3}\right)-\left(3\times(-1)+\frac{2\times(-1)^2}{2}-\frac{(-1)^3}{3}\right)$	
	•⁴ evaluate integral	•⁴ $\frac{32}{3}$ (units²)	

Notes:

1. •¹ is not available to candidates who omit 'dx'.
2. Limits must appear at the •¹ stage for •¹ to be awarded.
3. Where a candidate differentiates one or more terms at •², then •³ and •⁴ are unavailable.
4. Candidates who substitute limits without integrating, do not gain •³ or •⁴.
5. Do not penalise the inclusion of '$+c$'.
6. Do not penalise the continued appearance of the integral sign after •¹.
7. If •⁴ is only given as a decimal then it must be given correct to 1 decimal place.

Commonly Observed Responses:

Candidate A		Candidate B	
$\int_{-1}^{3}3+2x-x^2$	•¹ ✗	$\int\left(3+2x-x^2\right)dx$	•¹ ✗
$=3x+\frac{2x^2}{2}-\frac{x^3}{3}$	•² ✓	$=3x+\frac{2x^2}{2}-\frac{x^3}{3}$	•² ✓
	•³ ^	$=9-\left(-\frac{5}{3}\right)$	•³ ✓
$=\frac{32}{3}$	•⁴ ✓ 1	$=\frac{32}{3}$	•⁴ ✓

Commonly Observed Responses:	
Candidate C $\int \left(3+2x-x^2\right)dx$ •1 ✘ $=3x+\frac{2x^2}{2}-\frac{x^3}{3}$ •2 ✓ $=\left(3\times3+\frac{2\times3^2}{2}-\frac{3^3}{3}\right)-\left(3\times(-1)+\frac{2\times(-1)^2}{2}-\frac{(-1)^3}{3}\right)$ •3 ✓ $=\frac{32}{3}$ •4 ✓	**Candidate D** $\int_3^{-1}\left(3+2x-x^2\right)dx$ •1 ✓ … •2 ✓•3 ✓ $=-\frac{32}{3}$, hence area is $\frac{32}{3}$ •4 ✓ However $-\frac{32}{3}=\frac{32}{3}$ does not gain •4.

<table>
<tr><th>Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>2. (a)</td><td>•1 find $\mathbf{u.v}$</td><td>•1 24</td><td>1</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4">Commonly Observed Responses:</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td>(b)</td><td>•2 find $|\mathbf{u}|$
•3 find $|\mathbf{v}|$
•4 apply scalar product
•5 calculate angle</td><td>•2 $\sqrt{26}$
•3 $\sqrt{138}$
•4 $\cos\theta° = \frac{24}{\sqrt{26}\sqrt{138}}$
•5 $66 \cdot 38...°$ or $1 \cdot 16...$radians</td><td>4</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">1. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. Eg $\sqrt{-1^2+4^2-3^2}=\sqrt{26}$ or $\sqrt{-1^2+4^2+-3^2}=\sqrt{26}$, •2 is awarded.
2. •4 is not available to candidates who simply state the formula $\cos\theta° = \frac{\mathbf{u.v}}{|\mathbf{u}||\mathbf{v}|}$.
3. Accept answers which round to 66° or $1 \cdot 2$ radians (or 73·8 gradians).
4. Do not penalise the omission or incorrect use of units.
5. •5 is only available for a single angle.
6. For a correct answer with no working award 0/4.</td></tr>
<tr><td colspan="4">Commonly Observed Responses:</td></tr>
<tr><td colspan="2">Candidate A
$|\mathbf{u}| = \sqrt{26}$ •2 ✓
$|\mathbf{v}| = \sqrt{138}$ •3 ✓
$\frac{24}{\sqrt{26}\sqrt{138}}$ •4 ^
$\theta = 66 \cdot 38...°$ •5 $\boxed{✓ 1}$</td><td colspan="2"></td></tr>
</table>

Question	Generic scheme	Illustrative scheme	Max mark
3.	$\bullet^1$ differentiate	$\bullet^1$ $3x^2-7$	**3**
	$\bullet^2$ evaluate derivative at $x=2$	$\bullet^2$ 5	
	$\bullet^3$ interpret result	$\bullet^3$ $(f \text{ is})$ increasing	

Notes:

1. $\bullet^2$ and $\bullet^3$ are only available as a consequence of working with a derivative.
2. Accept $f'(2)>0$ for $\bullet^2$.
3. $f'(x)>0$ with no evidence of evaluating the derivative at $x=2$ does not gain $\bullet^2$ or $\bullet^3$. See candidate B.
4. Do not penalise candidates who use y in place of $f(x)$.

Commonly Observed Responses:

Candidate A

$3x^2-7$ $\bullet^1$ ✓

x	2
$f'(x)$	$+$

$\bullet^2$ ✓

increasing $\bullet^3$ ✓

Candidate B

$3x^2-7$ $\bullet^1$ ✓

$f'(x)>0$ $\bullet^2$ ^

f is increasing $\bullet^3$ ^

Question	Generic scheme	Illustrative scheme	Max mark
4.	**Method 1**	**Method 1**	3
	•¹ identify common factor	•¹ $-3\left(x^2+2x\ldots\right.$ stated or implied by •²	
	•² complete the square	•² $-3(x+1)^2\ldots$	
	•³ process for c	•³ $-3(x+1)^2+10$	
	Method 2	**Method 2**	
	•¹ expand completed square form	•¹ $ax^2+2abx+ab^2+c$	
	•² equate coefficients	•² $a=-3,\ 2ab=-6\ \ ab^2+c=7$	
	•³ process for b and c and write in required form	•³ $-3(x+1)^2+10$	

Notes:

1. $-3(x+1)^2+10$ with no working gains •¹ and •² only; however, see Candidate E.
2. •³ is only available for a calculation involving both multiplication and addition of integers.

Commonly Observed Responses:

Candidate A		Candidate B	
$-3\left(x^2+2\right)+7$	exception in General marking principle (h)	$-3\left(\left(x^2-6x\right)+7\right)$	•¹ ✗
$-3\left(\left(x+1\right)^2-1\right)+7$	•¹ ✓ •² ✓	$-3\left(\left(x-3\right)^2-9\right)+7$	•² $\boxed{✓\ 1}$
$-3(x+1)^2+10$	•³ ✓	$-3(x-3)^2+34$	•³ $\boxed{✓\ 1}$

Candidate C		Candidate D	
$a(x+b)^2+c=ax^2+2abx+ab^2+c$	•¹ ✓	$ax^2+2abx+ab^2+c$	•¹ ✓
$a=-3,\quad 2ab=-6,\quad ab^2+c=7$	•² ✓	$a=-3,\quad 2ab=-6,\quad ab^2+c=7$	•² ✓
$b=1,\quad c=10$	•³ ✓	$b=1,\quad c=10$	•³ ✗
•³ is awarded as all working relates to completed square form		•³ is lost as no reference is made to completed square form	

Commonly Observed Responses:	
Candidate E $-3(x+1)^2+10$ Check: $=-3(x^2+2x+1)+10$ $=-3x^2-6x-3+10$ $=-3x^2-6x+7$ Award 3/3	**Candidate F** $-3x^2-6x+7$ $=-3(x+1)^2-1+7$ •1 ✓ •2 ✓ $=-3(x+1)^2+6$ •3 ✗
Candidate G $-3x^2-6x+7$ $=x^2+2x-\frac{7}{3}$ •1 ✗ $=(x+1)^2-\frac{10}{3}$ •2 ✗ $=-3(x+1)^2+10$ •3 ✗	

Question	Generic scheme	Illustrative scheme	Max mark
5. (a)	$\bullet^1$ find the midpoint of PQ $\bullet^2$ calculate m_{PQ} and state perp. gradient $\bullet^3$ find equation of L_1 in a simplified form	$\bullet^1$ $(6,1)$ $\bullet^2$ $-1 \Rightarrow m_{\text{perp}} = 1$ $\bullet^3$ $y = x - 5$	**3**

Notes:

1. $\bullet^3$ is only available as a consequence of using a perpendicular gradient and a midpoint.
2. The gradient of the perpendicular bisector must appear in simplified form at $\bullet^2$ or $\bullet^3$ stage for $\bullet^3$ to be awarded.
3. At $\bullet^3$, accept $x - y - 5 = 0$, $y - x = -5$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(b)	$\bullet^4$ determine y coordinate $\bullet^5$ state x coordinate	$\bullet^4$ 5 $\bullet^5$ 10	**2**

Notes:

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
(c)	$\bullet^6$ calculate radius of the circle $\bullet^7$ state equation of the circle	$\bullet^6$ $\sqrt{50}$ stated or implied by $\bullet^7$ $\bullet^7$ $(x-10)^2 + (y-5)^2 = 50$	**2**

Notes:

4. Where candidates have calculated the coordinates of C incorrectly, $\bullet^6$ and $\bullet^7$ are available for using either PC or QC for the radius.
5. Where incorrect coordinates for C appear without working, only $\bullet^7$ is available.
6. Do not accept $\left(\sqrt{50}\right)^2$ for $\bullet^7$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)	(i)	●[1] start composite process ●[2] substitute into expression	●[1] $f(2x)$ ●[2] $3+\cos 2x$	**2**
		(ii)	●[3] state second composite	●[3] $2(3+\cos x)$	**1**

Notes:

1. For $3+\cos 2x$ without working, award both ●[1] and ●[2].
2. Candidates who interpret the composite function as either $g(x)\times f(x)$ or $g(x)+f(x)$ do not gain any marks.

Commonly Observed Responses:

Candidate A - interpret $f(g(x))$ as $g(f(x))$		**Candidate B - interpret $f(g(x))$ as $g(f(x))$**	
(i) $2(3+\cos x)$	●[1] ✗ ●[2] $\boxed{✓ 1}$	(i) $f(2x)=2(3+\cos x)$	●[1] ✓ ●[2] ✗
(ii) $3+\cos 2x$	●[3] $\boxed{✓ 1}$	(ii) $3+\cos(2x)$	●[3] $\boxed{✓ 1}$

Question	Generic scheme	Illustrative scheme	Max mark
6. (b)	•[4] equate expressions from (a)	•[4] $3+\cos 2x = 2(3+\cos x)$	**6**
	•[5] substitute for $\cos 2x$ in equation	•[5] $3+2\cos^2 x - 1 = 2(3+\cos x)$	
	•[6] arrange in standard quadratic form	•[6] $2\cos^2 x - 2\cos x - 4 = 0$	
	•[7] factorise	•[7] $2(\cos x - 2)(\cos x + 1)$	
	•[8] solve for $\cos x$	•[8] (•[8]) $\cos x = 2$ (•[9]) $\cos x = -1$	
	•[9] solve for x	•[9] $\cancel{\cos x = 2}$ or eg 'no solution' $x = \pi$	

Notes:

3. Do not penalise absence of common factor at •[7].
4. •[5] cannot be awarded until the equation reduces to a quadratic in $\cos x$.
5. Substituting $2\cos^2 A - 1$ or $2\cos^2 \alpha - 1$ at •[5] stage should be treated as bad form provided the equation is written in terms of x at •[6] stage. Otherwise, •[5] is not available.
6. '$=0$' must appear by •[7] stage for •[6] to be awarded. However, for candidates using the quadratic formula to solve the equation, '$=0$' must appear at •[6] stage for •[6] to be awarded.
7. For candidate who do not arrange in standard quadratic form, eg $-2\cos x + 2\cos^2 x - 4 = 0$ •[6] is only available if •[7] has been awarded.
8. •[7] •[8] and •[9] are only available as a consequence of solving a quadratic with distinct real roots.
9. •[7] •[8] and •[9] are not available for any attempt to solve a quadratic equation written in the form $ax^2 + bx = c$.
10. •[9] is not available to candidates who work in degrees and do not convert their solution(s) into radian measure.
11. Answers written as decimals should be rounded to no fewer than 2 significant figures.
12. •[9] is not available for any solution containing angles outwith the interval $0 \le x < 2\pi$.

Commonly Observed Responses:	
Candidate C **Quadratic expressed in terms of c or x.** $3+\cos 2x = 2(3+\cos x)$ ●4 ✓ $3+2\cos^2 x - 1 = 2(3+\cos x)$ ●5 ✓ $2\cos^2 x - 2\cos x - 4 = 0$ ●6 ✓ $2c^2 - 2c - 4 = 0$ $2(c-2)(c+1) = 0$ ●7 ✓ $c = 2$, $c = -1$ ●8 ✗ no solution, $x = \pi$ ●9 ✓ However, ●4 ✓ ●5 ✓ ●6 ✓ $2(c-2)(c+1) = 0$ ●7 ✓ $\cos x = 2$ $\cos x = -1$ ●8 ✓ Solution stated in terms of $\cos x$ explicitly	**Candidate D** $3+\cos 2x = 2(3+\cos x)$ ●4 ✓ $3+2\cos^2 x - 1 = 2(3+\cos x)$ ●5 ✓ $2\cos^2 x - 2\cos x = 4$ ●6 ^ $\cos^2 x - \cos x = 2$ $\cos x(\cos x - 1) = 2$ ●7 $\boxed{✓\ 2}$ $\cos x = 2,\ \cos x - 1 = 2$ $\cos x = 2,\ \cos x = 3$ ●8 ✗ no solutions ●9 ✗ see note 9
Candidate E - reading $\cos 2x$ as $\cos^2 x$ $3+\cos^2 x = 2(3+\cos x)$ ●4 ✗ ●5 ^ - no substitution required $\cos^2 x - 2\cos x - 3 = 0$ ●6 $\boxed{✓\ 1}$ $(\cos x - 3)(\cos x + 1)$ ●7 $\boxed{✓\ 1}$ $\cos x = 3,\ \cos x = -1$ ●8 $\boxed{✓\ 1}$ no solution, $x = \pi$ ●9 $\boxed{✓\ 1}$	**Candidate F - using quadratic formula** 4 ✓ ●5 ✓ $2\cos^2 x - 2\cos x - 4 = 0$ ●6 ✓ $\cos x = \dfrac{2 \pm \sqrt{36}}{4}$ or $\cos x = \dfrac{1 \pm \sqrt{9}}{2}$ ●7 ✓

Question	Generic scheme	Illustrative scheme	Max mark
7. (a) (i)	•¹ use '2' in synthetic division or in evaluation of cubic	•¹ $\begin{array}{r\|rrrr} 2 & 2 & -3 & -3 & 2 \\ & & & & \\ \hline & 2 & & & \end{array}$ or $2\times(2)^3-3(2)^2-3\times(2)+2$	**2**
	•² complete division/evaluation and interpret result	•² $\begin{array}{r\|rrrr} 2 & 2 & -3 & -3 & 2 \\ & & 4 & 2 & -2 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$ Remainder $=0 \therefore (x-2)$ is a factor or $f(2)=0 \therefore (x-2)$ is a factor	
(ii)	•³ state quadratic factor	•³ $2x^2+x-1$	**2**
	•⁴ complete factorisation	•⁴ $(x-2)(2x-1)(x+1)$ stated explicitly	

Notes:

1. Communication at •² must be consistent with working at that stage i.e. a candidate's working must arrive legitimately at 0 before •² can be awarded.
2. Accept any of the following for •² :
   - '$f(2)=0$ so $(x-2)$ is a factor'
   - 'since remainder = 0, it is a factor'
   - the 0 from any method linked to the word 'factor' by e.g. 'so', 'hence', '$\therefore$', '$\rightarrow$', '$\Rightarrow$'
3. Do not accept any of the following for •²:
   - double underlining the zero or boxing the zero without comment
   - '$x=-2$ is a factor', '$(x+2)$ is a factor', '$(x+2)$ is a root', '$x=2$ is a root', '$(x-2)$ is a root', '$x=-2$ is a root'
   - the word 'factor' only, with no link.

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
7. (b)	•⁵ demonstrate result	•⁵ $u_6=a(2a-3)-1=2a^2-3a-1$ leading to $u_7=a(2a^2-3a-1)-1$ $=2a^3-3a^2-a-1$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(c)	(i)	•6 equate u_5 and u_7 and arrange in standard form •7 solve cubic •8 discard invalid solutions for a	•6 $2a^3-3a^2-3a+2=0$ •7 $a=2,\ a=\frac{1}{2},\ a=-1$ •8 $a=\frac{1}{2}$	3
		(ii)	•9 calculate limit	•9 -2	1

Notes:

4. Where •6 has been awarded, •7 is available for solutions in terms of x appearing in a(ii). However, see Candidates B and C. BEWARE: Candidates who make a second attempt at factorising the cubic obtained in c(i) and do so incorrectly cannot be awarded mark 7 for solutions appearing in a(ii).
5. •8 is only available as a result of a valid strategy at •7.
6. $x=\frac{1}{2}$ does not gain •8.
7. For candidates who do not state the cubic equation at •6, and adopt a guess and check approach, using solutions for x found in a(ii), may gain 3/3. See Candidate D.

Commonly Observed Responses:

Candidate A

$2a^3-3a^2-3a+2=0$ •6 ✓

$x=2,\ x=\frac{1}{2},\ x=-1$ in a(ii) •7 ✓ •8 ^

Candidate B - missing '$=0$' from equation

$2a^3-3a^2-3a+2$ •6 ^

$\cancel{x=2},\ x=\frac{1}{2},\ \cancel{x=-1}$ in a(ii) •7 $\boxed{✓ 1}$

$a=\frac{1}{2}$ •8 $\boxed{✓ 1}$

Candidate C - missing '$=0$' from equation

$2a^3-3a^2-3a+2$ •6 ^

$\cancel{x=2},\ x=\frac{1}{2},\ \cancel{x=-1}$ in a(ii) •7 ^

$\frac{1}{2}$ •8 ^

No clear link between a and x.

Candidate D - $x=-1,\ x=\frac{1}{2}$ and $x=2$ identified in a(ii)

$u_5=2\left(\frac{1}{2}\right)-3=-2$ •6 ✓

$u_7=2\left(\frac{1}{2}\right)^3-3\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)-1=-2$ •7 ✓

$a=\frac{1}{2}$ because $-1<a<1$ •8 ✓

Question		Generic scheme	Illustrative scheme	Max mark
8.	(a)	•1 use compound angle formula	•1 $k\cos x^\circ \cos a^\circ + k\sin x^\circ \sin a^\circ$ stated explicitly	**4**
		•2 compare coefficients	•2 $k\cos a^\circ = 2$ and $k\sin a^\circ = -1$ stated explicitly	
		•3 process for k	•3 $k = \sqrt{5}$	
		•4 process for a and express in required form	•4 $\sqrt{5}\cos\left(x - 333\cdot 4\ldots\right)^\circ$	

Notes:

1. Accept $k\left(\cos x^\circ \cos a^\circ + \sin x^\circ \sin a^\circ\right)$ for •1. Treat $k\cos x^\circ \cos a^\circ + \sin x^\circ \sin a^\circ$ as bad form only if the equations at the •2 stage both contain k.
2. Do not penalise the omission of degree signs.
3. $\sqrt{5}\cos x^\circ \cos a^\circ + \sqrt{5}\sin x^\circ \sin a^\circ$ or $\sqrt{5}\left(\cos x^\circ \cos a^\circ + \sin x^\circ \sin a^\circ\right)$ is acceptable for •1 and •3.
4. •2 is not available for $k\cos x^\circ = 2$, $k\sin x^\circ = -1$, however •4 may still be gained.
5. •3 is only available for a single value of k, $k > 0$.
6. •4 is not available for a value of a given in radians.
7. Accept any value of a which rounds to 333°
8. Candidates may use any form of the wave function for •1, •2 and •3, however, •4 is only available if the wave is interpreted in the form $k\cos\left(x - a\right)^\circ$.
9. Evidence for •4 may not appear until part (b).

Commonly Observed Responses:

Responses with missing information in working:

Candidate A

•1 ^

$\sqrt{5}\cos a^\circ = 2$

$\sqrt{5}\sin a^\circ = -1$ •2 ✓ •3 ✓

$\tan a^\circ = -\frac{1}{2}$

$a = 333\cdot 4$

$\sqrt{5}\cos\left(x - 333\cdot 4\right)^\circ$ •4 ✓

Candidate B

$k\cos x^\circ \cos a^\circ + k\sin x^\circ \sin a^\circ$ •1 ✓

$\cos a^\circ = 2$

$\sin a^\circ = -1$ •2 ✗

$\tan a^\circ = -\frac{1}{2}$

$a = 333\cdot 4$

Not consistent with equations at •2.

$\sqrt{5}\cos\left(x - 333\cdot 4\right)^\circ$ •3 ✓ •4 ✗

Candidate C

$\cos x^\circ \cos a^\circ + \sin x^\circ \sin a^\circ$ •1 ✗

$\cos a^\circ = 2$

$\sin a^\circ = -1$ •2 ✗

$k = \sqrt{5}$ •3 ✓

$\tan a^\circ = -\frac{1}{2}$

$a = 333\cdot 4$

$\sqrt{5}\cos\left(x - 333\cdot 4\right)^\circ$ •4 ✗

Responses with the correct expansion of $k\cos(x-a)°$ but errors for either $\bullet^2$ or $\bullet^3$:		
Candidate D $k\cos x°\cos a° + k\sin x°\sin a°$ $\bullet^1$ ✓ $k\cos a° = 2$ $k\sin a° = -1$ $\bullet^2$ ✓ $\tan a° = -2$ $\bullet^3$ ^ $\bullet^4$ ✗ $a = 296\cdot6$	**Candidate E** $k\cos x°\cos a° + k\sin x°\sin a°$ $\bullet^1$ ✓ $k\cos a° = -1$ $k\sin a° = 2$ $\bullet^2$ ✗ $\tan a° = -2$ $a = 116\cdot6$ $\sqrt{5}\cos(x-116\cdot6)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{\checkmark\ 1}$	**Candidate F** $k\cos x°\cos a° + k\sin x°\sin a°$ $\bullet^1$ ✓ $k\cos a° = 2$ $k\sin a° = 1$ $\bullet^2$ ✗ $\tan a° = \frac{1}{2}$ $a = 26\cdot6$ $\sqrt{5}\cos(x-26\cdot6)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{\checkmark\ 1}$
Commonly Observed Responses:		
Responses with the incorrect labelling, $k(\cos A\cos B + \sin A\sin B)$ from the formula list:		
Candidate G $k\cos A\cos B + k\sin A\sin B$ $\bullet^1$ ✗ $k\cos a° = 2$ $k\sin a° = -1$ $\bullet^2$ ✓ $\tan a° = -\frac{1}{2}$ $a = 333\cdot4$ $\sqrt{5}\cos(x-333\cdot4)°$ $\bullet^3$ ✓ $\bullet^4$ ✓	**Candidate H** $k\cos A\cos B + k\sin A\sin B$ $\bullet^1$ ✗ $k\cos x° = 2$ $k\sin x° = -1$ $\bullet^2$ ✗ $\tan x° = -\frac{1}{2}$ $x = 333\cdot4$ $\sqrt{5}\cos(x-333\cdot4)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{\checkmark\ 1}$	**Candidate I** $k\cos A\cos B + k\sin A\sin B$ $\bullet^1$ ✗ $k\cos B° = 2$ $k\sin B° = -1$ $\bullet^2$ ✗ $\tan B° = -\frac{1}{2}$ $B = 333\cdot4$ $\sqrt{5}\cos(x-333\cdot4)°$ $\bullet^3$ ✓ $\bullet^4$ $\boxed{\checkmark\ 1}$

Question			Generic scheme	Illustrative scheme	Max mark
8.	(b)	(i)	•5 state minimum value	•5 $-3\sqrt{5}$ or $-\sqrt{45}$	1
		(ii)	**Method 1** •6 start to solve •7 state value of x **Method 2** •6 start to solve •7 state value of x	**Method 1** •6 $x-333\cdot4=180$ leading to $x=513\cdot4$ •7 $x=153\cdot4\ldots$ **Method 2** •6 $x-333\cdot4=-180$ •7 $x=153\cdot4\ldots$	2

Notes:

10. •7 is only available for a single value of x.
11. •7 is only available in cases where $a<-180$ or $a>180$. See Candidate J

Commonly Observed Responses:

Candidate J - from $\sqrt{5}\cos(x-26\cdot6)^\circ$ $x-26\cdot6=180$ $x=206\cdot6$ •6 ✓ 1 •7 ✓ 2 Similarly for $\sqrt{5}\cos(x-116\cdot6)^\circ$	**Candidate K - from 'minimum' of eg $-\sqrt{5}$** $3\sqrt{5}\cos(x-333\cdot4)^\circ=-\sqrt{5}$ $\cos(x-333\cdot4)^\circ=-\frac{1}{3}$ $x-333\cdot4=109\cdot5,\ 250\cdot5$ $x=442\cdot9,\ 583\cdot9$ •6 ✓ 1 $x=82\cdot9,\ 223\cdot9$ •7 ✗

Question	Generic scheme	Illustrative scheme	Max mark
9.	$\bullet^1$ express P in differentiable form $\bullet^2$ differentiate $\bullet^3$ equate expression for derivative to 0 $\bullet^4$ process for x $\bullet^5$ verify nature $\bullet^6$ evaluate P	$\bullet^1$ $2x+128x^{-1}$ $\bullet^2$ $2-\dfrac{128}{x^2}$ $\bullet^3$ $2-\dfrac{128}{x^2}=0$ $\bullet^4$ 8 $\bullet^5$ table of signs for a derivative (see next page) $\therefore$ minimum or $P''(8)=\dfrac{1}{2}>0$ $\therefore$ minimum $\bullet^6$ $P=32$ or min value $=32$	6

Notes:

1. For a numerical approach award 0/6.
2. For candidates who integrate any term at the $\bullet^2$ stage, only $\bullet^3$ is available on follow through for setting their 'derivative' to 0.
3. $\bullet^4$, $\bullet^5$ and $\bullet^6$ are only available for working with a derivative which contains an index ≤ -2.
4. At $\bullet^2$ accept $2-128x^{-2}$.
5. Ignore the appearance of -8 at $\bullet^4$.
6. $\sqrt{\dfrac{128}{2}}$ must be simplified at $\bullet^4$ or $\bullet^5$ for $\bullet^4$ to be awarded.
7. $\bullet^5$ is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 8.
8. $\bullet^6$ is still available in cases where a candidate's table of signs does not lead legitimately to a minimum at $\bullet^5$.
9. $\bullet^5$ and $\bullet^6$ are not available to candidates who state that the minimum exists at a negative value of x.

Commonly Observed Responses:

Candidate A - differentiating over more than one line	Candidate B - differentiating over more than one line
$\bullet^1$ ^ $P'(x)=2+128x^{-1}$ $P'(x)=2-128x^{-2}$ $\bullet^2$ ✗ $2-128x^{-2}=0$ $\bullet^3$ $\boxed{✓ 1}$	$P(x)=2x+128x^{-1}$ $\bullet^1$ ✓ $P'(x)=2+128x^{-1}$ $P'(x)=2-128x^{-2}$ $\bullet^2$ ✗ $2-128x^{-2}=0$ $\bullet^3$ $\boxed{✓ 1}$

Table of signs for a derivative

Accept:

x	-8^-	-8	-8^+
$P'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	8^-	8	8^+
$P'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

Here, for exemplification, tables of signs considering both roots separately have been displayed. However, in this question, it was only expected that candidates would consider the positive root for $\bullet^5$. Do not penalise the consideration of the negative root.

x	$\rightarrow$	-8	$\rightarrow$
$P'(x)$	$+$	0	$-$
Shape or Slope	/	—	\

x	$\rightarrow$	8	$\rightarrow$
$P'(x)$	$-$	0	$+$
Shape or Slope	\	—	/

Arrows are taken to mean 'in the neighbourhood of'

x	a	-8	b	c	8	d
$P'(x)$	$+$	0	$-$	$-$	0	$+$
Shape or Slope	/	—	\	\	—	/

Where: $a<-8$, $-8<b<0$, $0<c<8$, $d>8$

Do not Accept:

x	a	-8	b	8	c
$P'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-8<b<8$' is not acceptable.

$P'(x)$	$+$	0	$-$	0	$+$
Shape or Slope	/	—	\	—	/

Since the function is discontinuous '$-8 \rightarrow 8$' is not acceptable.

General Comments:

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $P'(x)$ in the table is an acceptable alternative to writing '$+$' or '$-$' signs. Values must be checked for accuracy.
    - The only acceptable variations of $P'(x)$ are: P', $\dfrac{dP}{dx}$ and $2-\dfrac{128}{x^2}$.

Question	Generic scheme	Illustrative scheme	Max mark
10.	•¹ use the discriminant •² identify roots of quadratic expression •³ apply condition •⁴ state range with justification	•¹ $(m-3)^2-4\times1\times m$ •² 1, 9 •³ $(m-3)^2-4\times1\times m>0$ •⁴ $m<1,\ m>9$ with eg sketch or table of signs	**4**

Notes:

1. If candidates have the condition '$\text{discriminant}<0$', '$\text{discriminant}\leq0$' or '$\text{discriminant}\geq0$', then •³ is lost but •⁴ is available.
2. Ignore the appearance of $b^2-4ac=0$ where the correct condition has subsequently been applied.
3. For candidates who have identified expressions for a, b, and c and then state $b^2-4ac>0$ award •³. See Candidate A.
4. For the appearance of x in any expression for •¹, award 0/4.

Commonly Observed Responses:

Candidate A

$(m-3)^2-4\times1\times m$	•¹ ✓
$m^2-10m+9=0$	
$m=1,\ m=9$	•² ✓
$b^2-4ac>0$	•³ ✓
$m<1,\ m>9$	•⁴ ^

Expressions for a, b, and c implied at •¹

Question	Generic scheme	Illustrative scheme	Max mark
11. (a)	•¹ substitute for P and t	•¹ $50 = 100\left(1 - e^{3k}\right)$	4
	•² arrange equation in the form $A = e^{kt}$	•² $0 \cdot 5 = e^{3k}$ or $-0 \cdot 5 = -e^{3k}$	
	•³ simplify	•³ $\ln 0 \cdot 5 = 3k$	
	•⁴ solve for k	•⁴ $k = -0 \cdot 231$	

Notes:

1. •² may be assumed by •³.
2. Any base may be used at •³ stage. See Candidate D.
3. Accept any answer which rounds to $-0 \cdot 2$.
4. •³ must be consistent with the equation of the form $A = e^{kt}$ at its first appearance.
5. For candidates whose working would (or should) arrive at $\log(\text{negative})$ •⁴ is not available.
6. Where candidates use a 'rule' masquerading as a law of logarithms, •³ and •⁴ is not available.

Commonly Observed Responses:

Candidate A		Candidate B	
$50 = 100\left(1 - e^{3k}\right)$	•¹ ✓	$0 \cdot 5 = 100\left(1 - e^{3k}\right)$	•¹ ✗
$0 \cdot 5 = -e^{3k}$	•² ✗	$0 \cdot 995 = e^{3k}$	•² $\boxed{\checkmark\ 1}$
$\ln(0 \cdot 5) = 3k$	•³ ✗	$\ln(0 \cdot 995) = 3k$	•³ $\boxed{\checkmark\ 1}$
$k = -0 \cdot 231$	•⁴ ✗	$k = -0 \cdot 0017\ldots$	•⁴ $\boxed{\checkmark\ 1}$
68·5	•⁵ $\boxed{\checkmark\ 1}$	$P = 0 \cdot 8319\ldots$	•⁵ $\boxed{\checkmark\ 1}$
31·5% still queueing	•⁶ $\boxed{\checkmark\ 1}$	99·2% still queuing	•⁶ $\boxed{\checkmark\ 1}$

Candidate C		Candidate D	
$50 = 100\left(1 - e^{3k}\right)$	•¹ ✓	$50 = 100\left(1 - e^{3k}\right)$	•¹ ✓
$-0 \cdot 5 = -e^{3k}$	•² ✓	$0 \cdot 5 = e^{3k}$	•² ✓
$\ln(-0 \cdot 5) = \ln\left(-e^{3k}\right)$	•³ ✗	$\log_{10}(0 \cdot 5) = 3k \log_{10} e$	•³ ✓
$k = -0 \cdot 231$	•⁴ ✗	$k = -0 \cdot 231$	•⁴ ✓
68·5	•⁵ $\boxed{\checkmark\ 1}$		
31·5% still queueing	•⁶ $\boxed{\checkmark\ 1}$		

Question	Generic scheme	Illustrative scheme	Max mark
(b)	•⁵ evaluate P for $t = 5$	•⁵ 68·5	2
	•⁶ interpret result	•⁶ 31·5% still queueing	

Notes:

7. •⁵ and •⁶ are not available where $k \geq 0$.
8. •⁶ is only available where the value of P in •⁵ was obtained by substituting into an exponential expression.

Commonly Observed Responses:

Candidate D - $k = -0 \cdot 2$		
63·2	•⁵ ✓	
36·8% still queueing	•⁶ ✓	

Question	Generic scheme	Illustrative scheme	Max mark
12. (a) (i)	•¹ write down coordinates of centre	•¹ $(13,-4)$	**1**
(ii)	•² substitute coordinates and process for c	•² $13^2+(-4)^2+14\times13-22\times(-4)\ldots$ leading to $c=-455$	**1**
Notes:			
1. Accept $x=13,\ y=-4$ for •¹. 2. Do not accept $g=13,\ f=-4$ or $13,\ -4$ for •¹. 3. For those who substitute into $r=\sqrt{g^2+f^2-c}$, working to find r must be shown for •² to be awarded.			
Commonly Observed Responses:			
(b) (i)	•³ calculate two key distances •⁴ state ratio	•³ two from $r_2=25$, $r_1=10$ and $r_2-r_1=15$ •⁴ $3:2$ or $2:3$	**2**
(ii)	•⁵ identify centre of C_2 •⁶ state coordinates of P	•⁵ $(-7,11)$ or $\begin{pmatrix}-7\\11\end{pmatrix}$ •⁶ $(5,2)$	**2**
Notes:			
4. The ratio must be consistent with the working for r_2-r_1 5. Evidence for •³ may appear on a sketch. 6. For $3:2$ or $2:3$ with no working, award 0/2. 7. At •⁶, the ratio used to identify the coordinates of P must be consistent with the sizes of the circles in the original diagram for •⁶ to be available.			
Commonly Observed Responses:			
(c)	•⁷ state equation	•⁷ $(x-5)^2+(y-2)^2=1600$ or $x^2+y^2-10x-4y-1571=0$	**1**
Notes:			
Commonly Observed Responses:			

[END OF MARKING INSTRUCTIONS]